Lecture 1: The Planck scale

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Role of mathematics

Mathematics and physics

- A common history
- Separation

Mathematical theories: what use are axioms?

- General proofs
- Conceptual reasoning

Hope and glory

Conjecture vs achievement

- Calculus
- Dirac delta
- Positrons
- Yang-Mills-Higgs theory

Period between conjecture and confirmation. Or not.

Quantum Field Theory

. . . in Minkowski space

- Streater-Wightman axioms: no 4d examples
- Perturbation expansion does not define a QFT
- Practical consequences
- Gauge theory: the million dollar question
- Lattice gauge theory: continuum limit?

The continuum

Are there modes of infinitely high energy?

- QFT yes, but new physics beyond the Planck scale
- ... or no physics beyond the Planck scale?
- Renormalisable QFT: can ignore problem

But gravity is maybe non-renormalisable...

The Planck scale

Quantum gravity with matter

$$m_{PL} = \sqrt{\frac{\hbar c}{G}} = 10^{19}\,\mathrm{GeV}/c^2$$

$$l_{PL} = \sqrt{\frac{\hbar G}{c^3}} = 10^{-35}\mathrm{m}$$

- Some massless QFTs have a scale invariance. $\hbar = ML$
- GR has a scale invariance. $G = L/M$
- QG-without-matter has a scale invariance. $G\hbar = L^2$
- QG with matter has no scale invariance. G and $\hbar$

$e^{i/G\hbar \int R}$

Evidence for new physics at high energy

. . . in Minkowski space

- Gauge unification
- Neutrino scale - see-saw
- NCG: fermion mass relation

c.f. elasticity $\longrightarrow$ atoms, Fermi $\longrightarrow$ electro-weak

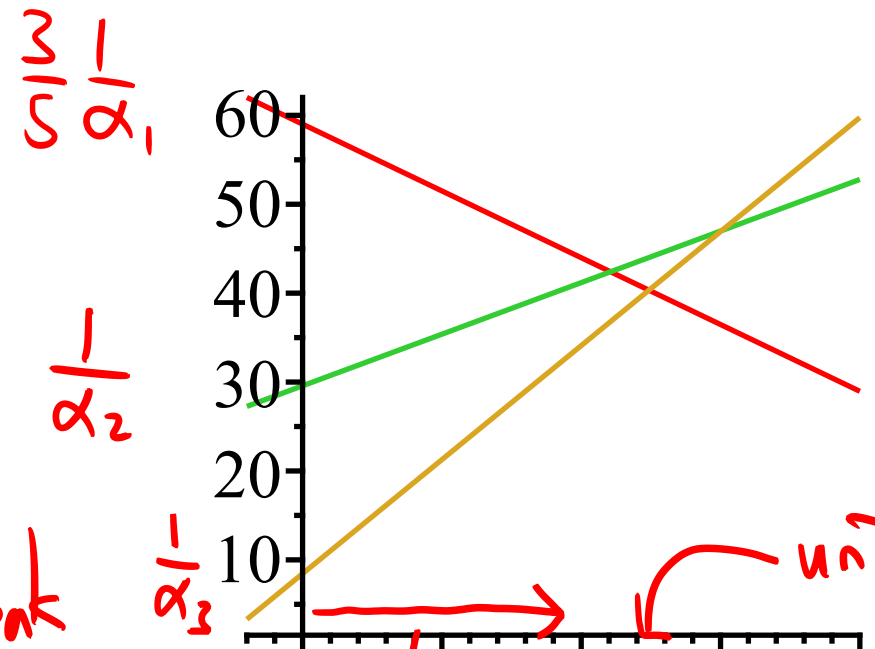
Gauge unification?

Running gauge couplings: big desert plus what?

Gauge couplings:

SU(3)	SU(2)	U(1)
g_3	g_2	g_1

$$\alpha_i = \frac{g_i^2}{4\pi} \qquad x = \log_{10}(\mu/M_Z)$$



$\frac{3}{5}\frac{1}{\alpha_1}$

$\frac{1}{\alpha_2}$

$\frac{1}{\alpha_3}$

Electro-Weak

unification?

Planck

V Scale

cosmic rays

Neutrino scale - see-saw

Cosmology: $\sum m < 0.3\text{eV}$

Oscillations: $\exists m > 0.04\text{eV}$

SM+ν_R mass matrix

$$\begin{pmatrix} \overline{\nu}_L & \nu_R \end{pmatrix} \begin{pmatrix} 0 & yH \\ yH & N \end{pmatrix} \begin{pmatrix} \overline{\nu}_L \\ \nu_R \end{pmatrix}$$

$$|\lambda| \simeq N,\ (yH)^2/N$$

$yH \sim 100\text{Gev},\ m_\nu \sim 0.01\text{ev} \quad \Rightarrow \quad N \sim 10^{15}\text{Gev}$

Standard model + ν_R + gravity

Fermions: $\Psi = 8 \times 3$ Dirac spinors

Bosons: $d =$ gravitational Dirac

$A =$ gauge fields

$H =$ Higgs

Generalised Dirac operator

$$D = d + A + yH + N$$

$y =$ Yukawa mass matrix

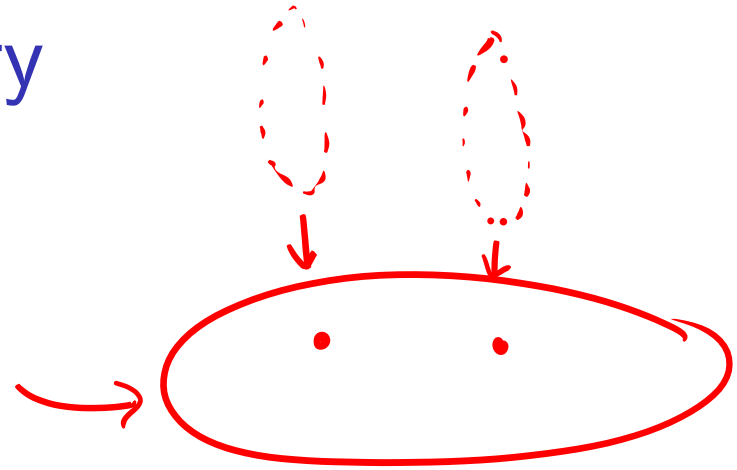
$N =$ Majorana mass matrix

Fermionic action

$$S = \int \overline{\Psi} D \Psi \, \mathrm{d}V$$

Fermionic SM

Non-commutative geometry

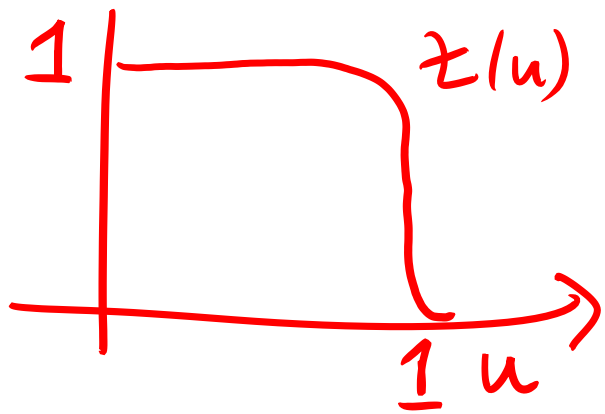


- Internal space is NC
- Space-time is commutative

Connes-Chamseddine spectral action

$$S_{CC} = \mathrm{Tr}\,\chi(D^2/l_{PL}^2)$$

Note: l_{PL} should read m_{PL}



- Euclidean
- Asymptotics $\rightarrow$ Bosonic SM + gravity

$l_{PL} \to \infty$

$S_{CC} \sim l_{PL}^4 \Lambda + l_{PL}^2 (\int R + \phi^2) + \int F^2 + (\partial\phi)^2 + \phi^4 + \ldots + l_{PL}^{-2}$

Non-commutative geometry predictions

1. At 'unification' scale

$$g_3 = g_2 = \sqrt{\frac{5}{3}} g_1$$

$$\sum_{\text{generations}} m_\nu^2 + m_e^2 + 3m_u^2 + 3m_d^2 = 8M_W^2$$

2. The gravitational action is

$$\int R - 2\Lambda + aRH^2 + bC^2$$

3. There is SM extension containing a neutral scalar field coupling to H^2.
4. The see-saw mechanism for neutrino masses

Beyond the Planck scale?

A particle can form a black hole if

Compton wavelength $<$ Schwarzschild radius

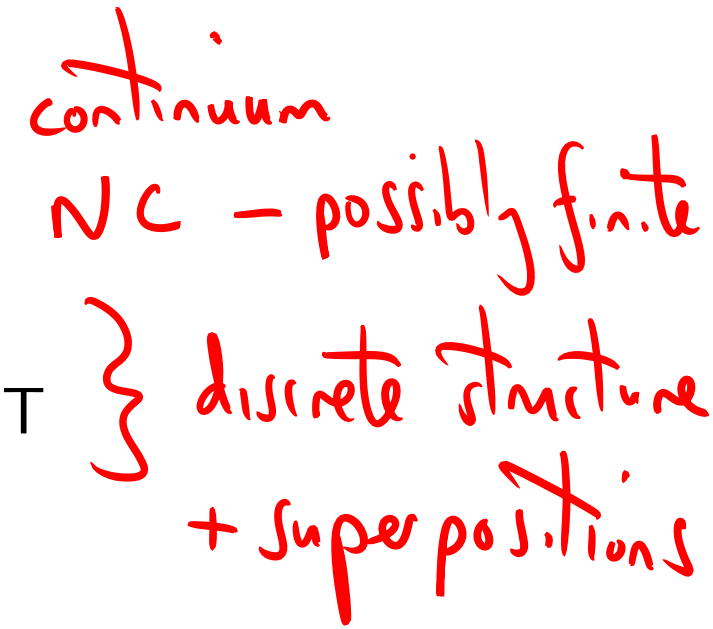
$$\frac{h}{m} < 2mG$$

i.e.,

$$m > \sqrt{\frac{\hbar}{4\pi G}} = \text{const. } m_{PL}$$

Models for the Planck scale

- QFT / Strings / Supergravity
- Non-commutative field theory
- Loop quantum gravity
- State sum models / spin foam / CDT



Lectures 2–5: current issues

Tools

- NCG
- State sum models

Questions

- Is there a NCG with Dirac operator for space-time?
- How to include fermionic matter in SSMs?
- Is there a role for NCG in SSM?